# WINTER BRAIDS XIII MONTPELLIER FEBRUARY 6–9, 2024

Time	Tuesday	Wednesday	Thursday	Friday
$     \begin{array}{r}                                     $	Registration	Petri 1	Golla 2	Golla 3
9:45 10:15	Bapat 1	Coffee break	Coffee break	Coffee break
$10:15 \\ 10:45$	Coffee break	Colla 1	Dotri 9	Dotri 2
$10:45 \\ 11:15$	Savk	Golla 1	reur 2	reur 5
11:15 - 13:15	Lunch break	Lunch break	Lunch break	Lunch break
13:15 14:15	Special lecture Pilaud	Bapat 2	Bapat 3	
$     \begin{array}{r}       14:30 \\       15:30     \end{array} $	Brejevs Smilga	Rodau Marino	Nanda Korinman	
$     \begin{array}{r}       15:30 \\       16:00     \end{array} $	Coffee break	Coffee break	Coffee break	
16:00 16:30	Flash Talks	Santoro	Bais	
10:30	Posters		Shah	

# Mini-courses and special lecture

# Asilata Bapat: Braids and Bridgeland stability conditions

The subject of this lecture series is categorical representations of braid groups. We introduce certain triangulated categories that carry a braid group action, from which we can recover known representations of the braid group on vector spaces. This "categorification" allows us to use new tools to study the interplay between the action of the braid group and the structure of the objects of the category.

One important tool, and the focus of this lecture series, is (Bridegland) stability conditions. Choosing a stability condition on a category is akin to choosing a basis for a vector space: one can study decompositions of objects of the category along the chosen stability condition. We will first introduce all these objects. We will then raise several (and answer some) questions about how stability conditions can be used to better understand the braid group, and viceversa.

## Marco Golla: Surfaces in 4-manifolds and complex curves

We will look at plane complex curves from a topological perspective, with the goal of restricting the possible configurations of singularities a complex curve of a given degree can have. The optimistic plan includes:

- a review on curves and their singularities;
- restrictions coming from branched covers of Riemann surfaces;
- restrictions coming from branched covers of complex surfaces (4-manifolds), using equivariant signatures and Tristram-Levine signatures.

# Bram Petri: Probabilistic methods in low-dimensional hyperbolic geometry

Hyperbolic manifolds are Riemannian manifolds whose metric is complete and has constant sectional curvature equal to -1. Another way to phrase the latter property is to say that they are locally isometric to hyperbolic space (of the right dimension). The geometry and topology of such manifolds plays a role in various domains of mathematics. In this mini course I will talk about how probabilistic methods can help to solve questions on the geometry and topology of hyperbolic manifolds.

## Vincent Pilaud: Subword complexes and brick polytopes

This talk will survey combinatorial, algebraic, and geometric aspects of subword complexes and brick polytopes. Based on various joint works with Nantel Bergeron, Noémie Cartier, Cesar Ceballos, Michel Pocchiola, Francisco Santos, Christian Stump.

# Short talks

# <u>Valentina Bais</u>: An exotic smooth structure on a non-orientable 4-manifold via Gluck twisting

I will talk about a joint work with Rafael Torres, in which we construct an exotic smooth structure on a non-orientable 4-manifold via a cut and paste operation called Gluck twist. Such 4-manifold is the total space of the non-trivial and non-orientable 2-sphere bundle over the real projective plane.

### Vitalijs Brejevs: On the symmetric braid index of ribbon knots

It is an open question whether every ribbon knot is a symmetric union knot, i.e., whether it admits a so-called symmetric union diagram that results from an appealing construction originally due to Kinoshita and Terasaka, subsequently generalised by Lamm. One reason why this question is challenging is that symmetric union knots so far have resisted a nondiagrammatic definition. In ongoing work with Feride Ceren Köse, we seek to investigate symmetric union knots by considering their braidings. In particular, we use Khovanov homology to show that there exist symmetric union knots whose symmetric braid index is strictly higher than the braid index; this has implications in view of the classification of finite concordance order 3-braid closures due to Lisca.

### Julien Korinman: Representations of stated skein algebras and applications

I will present new results concerning the classification of representations of stated skein algebras at roots of unity. Such indecomposable representations are conjectured to be the building blocks of some new TQFTs. As application, I will describe how we can define the links invariants of such TQFTs and how we can define some new mapping class groups projective representations.

#### <u>Laura Marino</u>: Computing the symmetric gl(1)-homology

Introduced by Robert and Wagner, the symmetric gl(n)-homologies provide a categorification of a certain family of Reshetikhin-Turaev polynomials. These homologies are difficult to compute in general, however in the case n=1 the symmetric gl(1)-homology admits a significantly simpler definition, and we can explicitly describe a basis for the spaces constituting its chain complex. In this talk we will present the gl(1)-construction and state some results obtained by computing the symmetric gl(1)-homology for small knots.

#### <u>Neha Nanda</u>: Right-angled Artin groups are symmetric diagram groups

Guba and Sapir introduced a family of groups associated with semigroup presentations which are called symmetric and planar diagram groups. In this talk, we discuss the idea of how right-angled Artin groups can be seen as symmetric diagram groups in contrast to the fact that not all right-angled Artin groups are planar diagram groups. This was motivated by the example of pure virtual planar braid groups being symmetric diagram groups. This is work in collaboration with Paolo Bellingeri and Anthony Genevois.

#### Adrien Rodau: Homology inclusion of line arrangements

We introduce a new topological invariant of complex line arrangements in  $\mathbb{CP}^2$ , which form a subfamily of complex algebraic curves. The main motivation of their study is to identify Zariski pairs of arrangements which have the same combinatorics but different embeddings. Building on ideas developed by B. Guerville-Ballé and W. Cadiegan-Schlieper, we consider the inclusion map of boundary manifold to the exterior and its effect on homology classes. A careful study of the graph Waldhausen structure of the boundary manifold allows to identify specific generators of the homology. Their potential images are encoded in a group, the graph stabiliser, with a nice combinatorial presentation. The invariant related to the inclusion map is an element of this group. Using a computer implementation and the braid monodromy, we compute the invariant for some examples and exhibit new ordered Zariski pairs.

#### Diego Santoro: Taut foliations from knot diagrams

Taut foliations have been a classical object of study in 3-manifolds theory. Recently, new interest in them has come from the investigation of the so-called "L-space conjecture", that predicts that Heegaard Floer L-spaces can be characterised as those 3-manifolds that do not admit coorientable taut foliations. A possible approach to the study of this conjecture is by analysing surgeries on knots and links. Most of the techniques employed for constructing taut foliations on Dehn surgeries usually make use of some special property of the exterior of the link (e.g. fiberedness). In this talk I will describe a procedure for constructing taut foliations that only makes use of diagrammatic properties of the knot.

# Oguz Savk: Embeddings of Brieskorn spheres in sums of complex projective planes

We will discuss smooth embeddings of Brieskorn spheres in sums of complex projective planes from both constructive and obstructive perspectives. Our results are based on Kirby calculus and Floer and gauge theoretic cobordism invariants.

### Tanushree Shah: On The Cost Function Associated With Legendrian Knots

In this article, we introduce a non-negative integer valued function that measures the obstruction for converting any topological isotopy between two Legendrian knots into a Legendrian isotopy. We refer to this function as Cost function. We show that the Cost function induces a metric on the set of topologically isotopic Legendrian knots. Hence the set of topologically isotopic Legendrian knots can be seen as a graph with path-metric given by the Cost function. We compute the Cost function for torus knots (which are Legendrian simple) and some twist knots (which are Legendrian non-simple). We investigate the behaviour of Cost function under the connect sum operation.

### Ilia Smilga: Distribution of minimal surfaces in compact hyperbolic 3-manifolds

In a classical work, Bowen and Margulis proved the equidistribution of closed geodesics in any hyperbolic manifold. Together with Jeremy Kahn and Vladimir Marković, we asked ourselves what happens in a three-manifold if we replace curves by surfaces. The natural analog of a closed geodesic is then a minimal surface, as totally geodesic surfaces exist only very rarely. Nevertheless, it still makes sense (for various reasons, in particular to ensure uniqueness of the minimal representative) to restrict our attention to surfaces that are almost totally geodesic. The statistics of these surfaces then depend very strongly on how we order them: by genus, or by area. If we focus on surfaces whose **area** tends to infinity, we conjecture that they do indeed equidistribute; we proved a partial result in this direction. If, however, we focus on surfaces whose **genus** tends to infinity, the situation is completely opposite: we proved that they then accumulate onto the totally geodesic surfaces of the manifold (if there are any).